

B.5 Linear Functions

Essential Question How can you describe the graph of an equation of the form $y = mx + b$?

The Meaning of a Word ● Function

Your score on a test is usually a **function** of how much you study.
If you study a lot, your score is usually high.
If you study a little, your score is usually low.



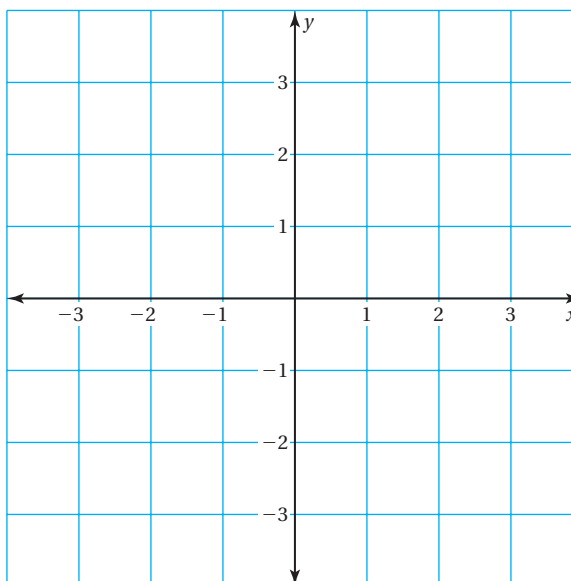
1 ACTIVITY: Using an Input-Output Table

Work with a partner.

- a. Copy and complete the input-output table for the equation $y = -\frac{1}{2}x + 2$.

| | | | | | | | |
|-------------|----|----|----|---|---|---|---|
| Input, x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| Output, y | | | | | | | |

- b. Plot the points from the table.



- c. Describe the pattern of the points. Draw a graph that represents the pattern.
- d. Choose three values of x that are not in the table. Find their corresponding y -values and plot the points. Do the points lie on the graph you made in part (c)?

Inductive Reasoning

Work with a partner. Sketch the graph of each equation. Then copy and complete the table.

| Equation | Description of Graph | Point of Intersection with y-axis | Slope of Graph |
|----------------------------|----------------------|-----------------------------------|----------------|
| 2. $y = -\frac{1}{2}x + 2$ | Line | (0, 2) | $-\frac{1}{2}$ |
| 3. $y = -x + 2$ | | | |
| 4. $y = -x + 1$ | | | |
| 5. $y = -\frac{1}{2}x + 1$ | | | |
| 6. $y = x + 1$ | | | |
| 7. $y = x - 1$ | | | |
| 8. $y = \frac{1}{2}x - 1$ | | | |
| 9. $y = \frac{1}{2}x + 1$ | | | |
| 10. $y = 2x + 1$ | | | |
| 11. $y = 2x - 2$ | | | |
| 12. $y = -2x + 3$ | | | |

What Is Your Answer?

13. **IN YOUR OWN WORDS** How can you describe the graph of an equation of the form $y = mx + b$?
- How does the value of m affect the graph?
 - How does the value of b affect the graph?
 - Test your answers to parts (a) and (b) with three equations that are not in the table.
14. Why is an equation of the form $y = mx + b$ called a linear function? What does the word *linear* mean? What does the word *function* mean?

Practice

Use what you learned about linear functions to complete Exercises 12–17 on page A40.

Key Vocabulary

linear function,
p. A38
y-intercept, p. A38
slope-intercept form,
p. A38

A **linear function** is a function whose graph is a line. The **y-intercept** is the y-coordinate of the point where the line crosses the y-axis.

Key Idea

Slope-Intercept Form

Words A linear function written in the form $y = mx + b$ is in **slope-intercept form**. The graph of the function is a line whose slope is m and whose y-intercept is b .

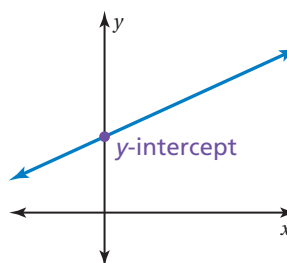
Algebra

$$y = mx + b$$

slope

y-intercept

Graph



EXAMPLE 1 Identifying Slopes and y-Intercepts

Find the slope and y-intercept of the graph of each function.

a. $y = -x - 5$

$$y = -1x + (-5)$$

Write in slope-intercept form.

∴ The slope is -1 and the y-intercept is -5 .

b. $y - 2 = -\frac{1}{3}x$

$$y = -\frac{1}{3}x + 2$$

Add 2 to each side.

∴ The slope is $-\frac{1}{3}$ and the y-intercept is 2.

c. $4y - 5x = 12$

$$4y = 5x + 12$$

Add 5x to each side.

$$y = \frac{5}{4}x + 3$$

Divide each side by 4.

∴ The slope is $\frac{5}{4}$ and the y-intercept is 3.

Common Error

The y-intercept of $y = -x - 5$ is not 5. Be sure to write equations in the form $y = mx + b$.

On Your Own

Find the slope and y -intercept of the graph of the linear function.

1. $y = -3x + 10$

2. $-5y + x = 25$

EXAMPLE 2 Graphing Lines Using Slope-Intercept Form

a. Graph $y = -2x + 3$.

Step 1: Find the slope and y -intercept.

$$y = -2x + 3$$

slope
y-intercept

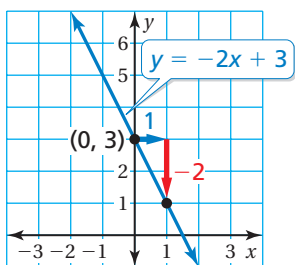
Step 2: The y -intercept is 3. So, plot $(0, 3)$.

Step 3: Find the rise and the run.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$$

Step 4: Plot the point that is **1 unit right** and **2 units down** from $(0, 3)$.

Step 5: Draw a line through the two points.



b. Graph $y = \frac{2}{3}x - 2$

Step 1: Find the slope and y -intercept.

$$y = \frac{2}{3}x + (-2)$$

slope
y-intercept

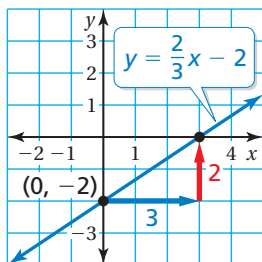
Step 2: The y -intercept is -2 . So, plot $(0, -2)$.

Step 3: Find the rise and the run.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{3}$$

Step 4: Plot the point that is **3 units right** and **2 units up** from $(0, -2)$.

Step 5: Draw a line through the two points.



On Your Own

Graph the linear function using slope-intercept form.

3. $y = -x - 1$

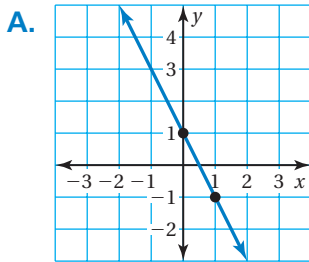
4. $y = \frac{3}{2}x - 5$

Vocabulary and Concept Check

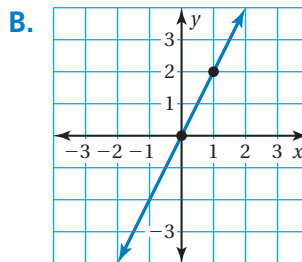
- VOCABULARY** What is the y -intercept of a line?
- WRITING** Why is $y = mx + b$ called the slope-intercept form of a line?

MATCHING Match the linear function with its graph.

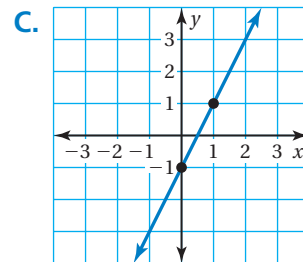
3. $y = 2x$



4. $y = 2x - 1$



5. $y = -2x + 1$



Practice and Problem Solving

Find the slope and y -intercept of the graph of the linear function.

1 6. $y = 4x + 1$

7. $y = -2x + 6$

8. $y = \frac{2}{3}x - 3$

9. $5x + y = 3$

10. $9x - 3y = 24$

11. $-4y + 10x = 36$

Graph the linear function using slope-intercept form.

2 12. $y = 3x - 3$

13. $y = 2x + 5$

14. $y = -x + 4$

15. $y = -\frac{3}{2}x - 1$

16. $y = -\frac{1}{5}x + 2$

17. $y = \frac{1}{4}x - 4$

18. $y = -4x + 1$

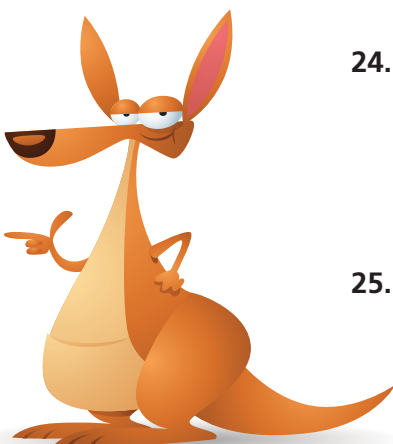
19. $y = 6x - 5$

20. $y = -3x - 2$

21. $5y - 4x = -15$

22. $5x + 3y = -6$

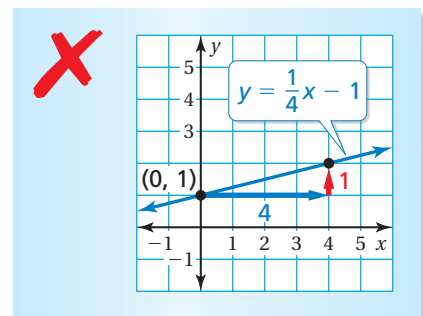
23. $3x + 4y = 12$



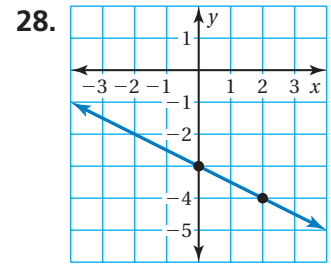
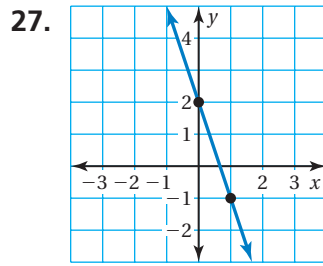
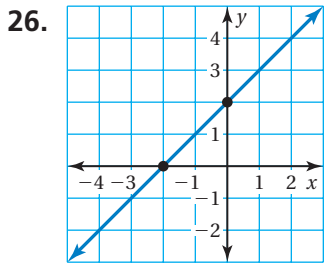
24. **ERROR ANALYSIS** Describe and correct the error in graphing the linear

function $y = \frac{1}{4}x - 1$.

25. **CARTOONIST** The number c of cartoons a cartoonist plans to complete by the n th day of the month is given by $c = 24 + 4n$. What does the y -intercept represent?

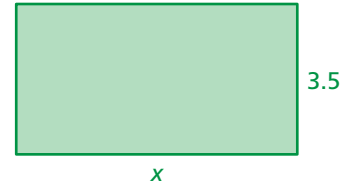


Write an equation of the linear function in slope-intercept form.



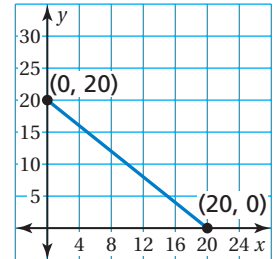
29. **PERIMETER** The perimeter of the rectangle can be modeled by the linear function $y = 2x + 7$.

- Find the slope and y -intercept of the graph of the linear function.
- Graph the linear function.
- Is it possible for the rectangle to have a perimeter of 5 units? Examine the graph and explain.



30. **ESCALATOR** To get from the second floor to the first floor in a mall, you can either ride the escalator or take the stairs. The graph shows the vertical distance y (in feet) you have left to travel on the escalator after x seconds.

- Write an equation in slope-intercept form of the linear function representing the amount of time you have left on the escalator.
- How long does it take to ride the escalator from the second floor to the first floor?
- The equation $y = -1.6x + 20$ represents the vertical distance y (in feet) you have left to travel on the stairs after x seconds. How much time do you save by taking the stairs?



31. **Reasoning** Find the slope and y -intercept of the graph of $Ax + By = C$ in terms of A , B , and C .



Fair Game Review what you learned in previous grades & lessons

Solve the equation.

32. $3x - 6 = 9 - 2x$

33. $5 - 8v = 3v - \frac{1}{2}$

34. $-5w - 4 = 4(w - 7)$

35. **MULTIPLE CHOICE** Which does *not* describe the sum of the sections of a circle graph?

(A) $\frac{1}{2}$

(B) 1

(C) 100%

(D) 360°